# A Scaling Law for Quark Masses

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**Abstract**

We show that the observed quark masses seem to be consistent with a simple scaling law. Due to the precise values of the heavy quarks we are able to calculate the quark masses in the light quark sector. We discuss a possible value for the strange quark mass. We show that the u-type quark masses obey the scaling law very well.

The masses of the quarks are important parameters of the Standard Model but thus far, have remained unexplained. In the Standard Model they are generated by the coupling of the quark fields to the hypothetical scalar boson, which breaks the $SU(2) \times U(1)$ symmetry.

The observed quark masses show a remarkable pattern. The $u$ and $d$ masses are relatively light, a few MeV in mass, the $s$ and $c$ masses are in the region of $100 \ldots 1200$ MeV, while the $b$ and $t$ masses are heavy. The t-mass of about 170 GeV is the only quark mass, which is of the same order as the energy scale describing the violation of the $SU(2)$ gauge symmetry.

In this paper we would like to argue that the quark masses might follow a simple scaling law:

$$\frac{m_t}{m_c} = \frac{m_c}{m_u}$$
$$\frac{m_b}{m_s} = \frac{m_s}{m_d} \tag{1}$$

Let us first demonstrate that the observed masses of the quarks might actually be consistent with the simple scaling laws. Of course, such laws make sense only, if the quark masses are all renormalized at the same energy scale. For the u-type-quarks we choose the central value of $m_t = 174.3 \pm 5.1$ GeV

as a useful scale. We choose for the c-quark mass $m_c(m_c) = 1.27 \pm 0.05$ GeV given in [1], which rescales to $m_c(m_t) = 0.62 \pm 0.03$ GeV, using the QCD renormalization group [2] with $\Lambda = 211^{+34}_{-30}$ MeV for five flavors [3]. Then one finds

$$\frac{m_t}{m_c} = 260 \ldots 304 \tag{2}$$

The u-mass $m_u$ is given as $m_u(1\,\mathrm{GeV}) = 5.1 \pm 0.9$ MeV [4]. Using the QCD renormalization group with $\Lambda = 211^{+34}_{-30}$ MeV for five flavors, one has $m_u(m_t) = 2.28 \pm 0.41$ MeV. Then we obtain:

$$\frac{m_c}{m_u} = 220 \ldots 348 \tag{3}$$

Both ratios are of the same order of magnitude. If they are set to be equal, we find for the central value of the mass ratios:

$$\frac{m_t}{m_c} = \frac{m_c}{m_u} = 281 \tag{4}$$

Thus we obtain

$$\begin{array}{rr} m_c(m_c) = 1.27 \text{ GeV} & m_c(m_t) = 0.62 \text{ GeV} \\ m_u(2\,\mathrm{GeV}) = 3.94 \text{ MeV} & m_u(m_t) = 2.21 \text{ MeV} \end{array} \tag{5}$$

Here the charm quark mass $m_c(m_c) = 1.27$ GeV is consistent with $m_c(m_c)$ $= 1.23 \pm 0.09$ GeV calculated from QCD sum rules in the charmonium system given in [5]. Indeed the top and charm quark are among the "heavy quarks", and their masses are known within small error bars [1]. Therefore the scaling law prediction for the up-quark mass is quite definitive. The error in the mass of u-quark stems from the error in the charm and top quark masses and depends also on the error in $\Lambda_{QCD}$ [3].

The same can be done for the d-type quarks. But among the d-type quarks $only$ the bottom quark is a "heavy quark" and has a relatively well known mass. On the contrary the error in the strange quark mass is rather high. Consequently the ratio $m_b/m_s$ will contain large uncertainties, and the scaling law prediction for the down quark mass will not be definitive. However using big error bars for the strange quark mass, it is still possible to have a consistent scaling law.

The scaling of the d-type quarks will be done at 2 GeV. For the bottom quark we choose $m_b(m_b) = 4.25 \pm 0.10$ [1] which rescales to $m_b(2\text{ GeV}) =$ $5.02 \pm 0.14$ GeV by using the QCD renormalization group with the current value $\Lambda = 294^{+42}_{-38}$ MeV, for 4 flavors given in "$\alpha_s$ 2002" [3]. We chose for

the strange quark mass $m_s(1\,\mathrm{GeV}) = 175 \pm 55$ MeV [1], which rescales to $m_s = 134 \pm 42$ MeV at 2 GeV by using the QCD renormalization group with the current value $\Lambda = 336^{+42}_{-38}$ MeV, for 3 flavors given in [3]. The down quark mass is chosen as $m_d(1\,\mathrm{GeV}) = 9.3 \pm 1.4$ MeV [4] which rescales to $m_d(2\ \mathrm{GeV}) = 7.1 \pm 1.1$ MeV. Then we obtain :

$$\frac{m_b}{m_s} = 28 \ldots 56 \qquad \frac{m_s}{m_d} = 11 \ldots 29 \tag{6}$$

which are again of the same order of magnitude. We can set them equal and find for the central values of the mass ratios:

$$\frac{m_b}{m_s} = \frac{m_s}{m_d} = 28 \tag{7}$$

Requiring the mass ratio to be the same, we have :

$$\begin{array}{ll} m_d(2\ \mathrm{Gev}) = 6.51\ \mathrm{MeV} & m_d(m_b) = 5.60\ \mathrm{MeV} \\ m_s(2\ \mathrm{Gev}) = 180\ \mathrm{MeV} & m_s(m_b) = 155\ \mathrm{MeV} \end{array} \tag{8}$$

Here $m_b(m_b) = 4.25$ GeV is consistent with an independent analysis giving $m_b(m_b) = 4.20 \pm 0.09$ GeV, derived from low-n sum rules in [6].

Thus we conclude that the observed quark masses seem to be consistent with the simple scaling laws. The best values for the ratio of the u-type quark masses is $281$ , for the d-type masses it is $28$. The quark masses that we found through the scaling law are consistent with current values of quark masses.

The strange quark mass from an analysis from the observed spectrum of $\tau$ decay [7] predicts $m_s = 170^{+44}_{-55}$ at 2 GeV and is consistent with the scaling law.

Of course in the Standard Model there is no reason for any scaling law. Such a reason can only be given in specific theories beyond the Standard Model. In this paper we do not wish to speculate about such reasons. However the fact that the scaling law discussed by us could be either approximately or perhaps even exactly true seems interesting and should be investigated further.

This Paper is dedicated to the memory of Prof. D. Tadic, who has contributed much to the investigation of the quark mass problem.

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